

Vectors and scalars

Vector and scalar quantities

When expressing a quantity, we give it a number and a unit (for example, 16 kg), this expresses the magnitude of the quantity. Some quantities also have direction. A quantity that has both a magnitude and direction is called a **vector**. On the other hand, a quantity that has only a magnitude is called a **scalar** quantity. Vectors are represented in print as bold and italicised characters (for example ***F***). Below is a table listing some vector and scalar quantities:

Scalars	Vectors
Distance	Displacement
Speed	Velocity
Energy	Acceleration
Temperature	Force
Distance	Momentum
Mass	

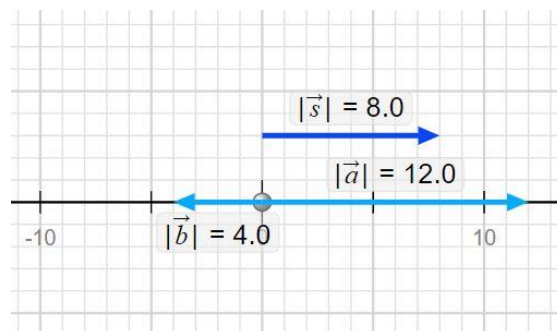
Examples of vector and scalar quantities

Note that some quantities appear to be the same, such as displacement and distance, both representing a distance, the difference is that displacement has a direction whilst distance does not. This also means that velocity is a vector (= displacement/time), whereas speed is a scalar (=distance/time).

Combining vectors by a graphical method

The difference of two vectors

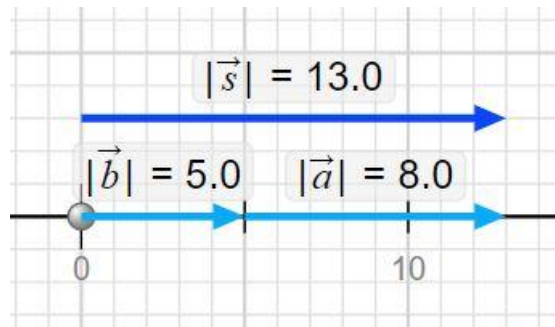
When adding vectors, we need to take both the magnitude and direction into account. Often, we will have situations where two vectors have opposite directions, in this case, we simply subtract the smallest magnitude from the largest one to find the resultant vector.



In this example, the resultant = $+12-4 = +8$

The sum of two vectors

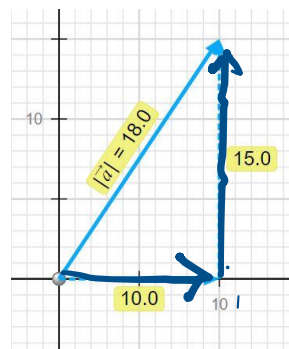
Sometimes we will have situations where two forces are acting in the same direction. In the situations we simply add together the magnitudes of both vectors.



$$\text{Resultant vector} = +5.0 + 8.0 = +13.0$$

Combining perpendicular vectors

We can combine perpendicular vectors and calculate the angle the resultant makes with either the horizontal or vertical. This can be done graphically, with a scale diagram, with the tail of one vector touching the head of the other. We then draw a line connecting the other head and tail. To get the magnitude of the new vector, we simply measure it and apply the same scaling. The angle can be measured with a protractor.

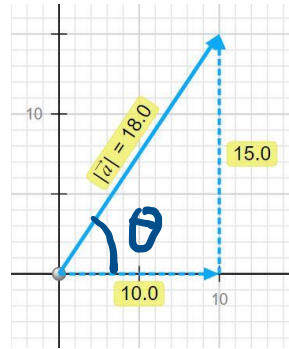


Combining perpendicular vectors by scale diagram

Alternatively, we can use trigonometry for a faster and more accurate result. In the above example, we can use pythagorus theorem to find the magnitude of the resultant:

$$\text{magnitude of resultant} = \sqrt{10^2 + 15^2} = 18.0$$

To find the angle of the resultant vector, we can draw the diagram as a triangle, which makes using trigonometry easier, then use an appropriate trig function:

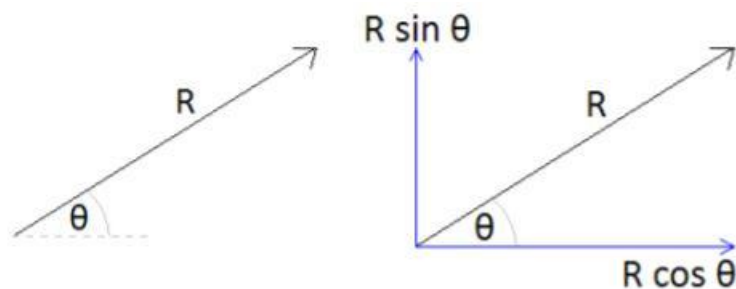


$$\text{Angle } \theta = \tan^{-1}(15/10) = 56.3^\circ$$

Resolving vectors into components

It is often helpful to resolve a vector into 2 perpendicular components. These components can then be combined with other vectors (of the same quantity) acting along the same line of action by simple addition.

Take vector R acting at an angle θ to the horizontal. The horizontal component is found from $R \cos \theta$ and the vertical component is found from $R \sin \theta$

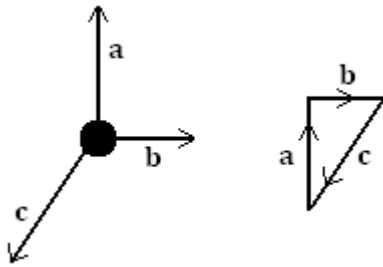


Resolving a vector into vertical and horizontal components

When working vectors that do not form a 90° angle, it is often useful to resolve certain vectors into component vectors so that they are in the same line of action as other vectors. To do this, we draw two vectors, one horizontal and the other vertical to our plane of reference.

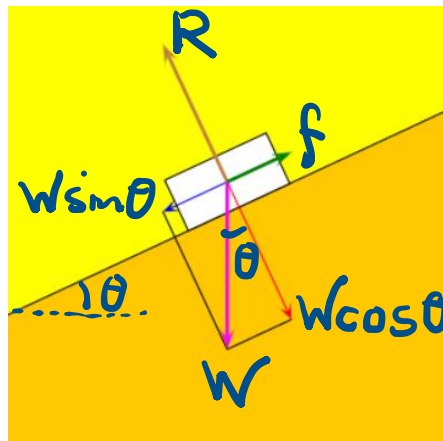
Equilibrium

When combining vectors with a scale diagram, if we end up in the same position we started at then equilibrium is achieved. We can resolve all vectors into their vertical and horizontal components. If the components up and down are equal and the components left and right are equal, equilibrium has been reached.



When we are considering the forces acting on an object, when all the forces acting on a body cancel out, equilibrium is reached and the object does not move (or maintains a steady velocity).

In the example of the inclined plane, the component of weight acting down the slope is $W\sin\theta$ and the component acting perpendicular to the slope is $W\cos\theta$.



Forces acting on a block on an inclined plane

If the block is in equilibrium, we can easily calculate the friction (=component of weight down slope) and the reaction force R acting on the block (=component of weight perpendicular to slope):

$$R = W \cos\theta$$

$$F = W \sin\theta$$