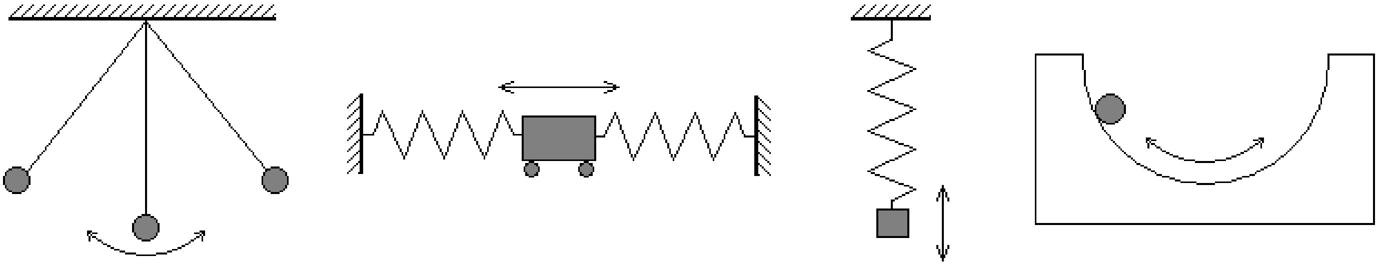
<u>Simple Harmonic Motion</u>

To know what simple harmonic motion is
To be able to describe the acceleration of an SHM system
To be able to calculate the displacement, velocity and acceleration of an SHM system

Oscillations

In each of the cases below there is something that is oscillating, it vibrates back and forth or up and down. Each of these systems is demonstrating Simple Harmonic Motion (SHM).



SHM Characteristics

The equilibrium point is where the object comes to rest, in the simple pendulum it at its lowest point. If we displace the object by a displacement of x there will be a force that brings the object back to the equilibrium point. We call this the restoring force and it always acts in the opposite direction to the displacement.

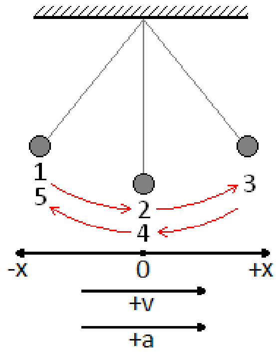
We can represent this as: $F \propto -x$

Since $F = ma$ we can also write: $a \propto -x$

For an object to be moving with simple harmonic motion, its acceleration must satisfy two conditions:

*The acceleration is proportional to the displacement
*The acceleration is in the opposite direction to the displacement (towards the equilibrium point)

Equations



The following equations are true for all SHM systems but let us use the simple pendulum when thinking about them.
The pendulum bob is displaced in the negative direction when at point 1, it is released and swings through point 2 at its maximum speed until it reaches point 3 where it comes to a complete stop. It then swings to the negative direction, reaches a maximum speed at 4 and completes a full cycle when it stops at 5.

Displacement, x

The displacement of the bob after a time t is given by the equation: $\boxed{x = A\cos 2\pi f t}$ **(CALCS IN RAD)**

Since $f = \frac{1}{T}$ the equation can become: $x = A\cos 2\pi \frac{1}{T} t$ → $x = A\cos 2\pi \frac{t}{T}$

(where t is the time into the cycle and T is the time for one complete cycle)
The maximum displacement is called the amplitude, A. $x = A$ ← MAXIMUM

Velocity, v

The velocity of the bob at a displacement of x is given by the equation: $\boxed{v = \pm 2\pi f \sqrt{A^2 - x^2}}$

The maximum velocity occurs in the middle of the swing (2 and 4) when displacement is zero (x = 0)

$v = \pm 2\pi f \sqrt{A^2 - x^2}$ → $v = \pm 2\pi f \sqrt{A^2 - 0^2}$ → $v = \pm 2\pi f \sqrt{A^2}$ → $v = \pm 2\pi f A$ ← MAXIMUM

Acceleration, a

The acceleration of the bob at a displacement of x is given by the equation: $\boxed{a = -(2\pi f)^2 x}$

As discussed before the acceleration acts in the opposite direction to the displacement.

The maximum acceleration occurs at the ends of the swing (1, 3 and 5) when the displacement is equal to the amplitude ($x = A$).

$a = -(2\pi f)^2 x$ → $a = -(2\pi f)^2 A$ ← MAXIMUM