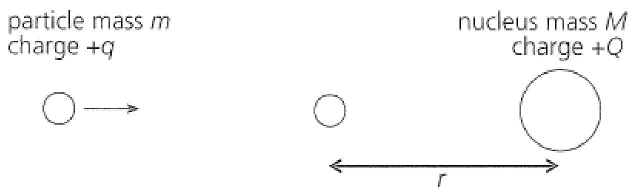
Nuclear Radius

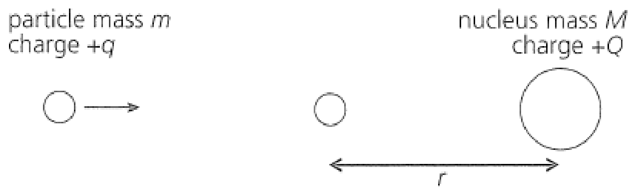
To be able to calculate the radius of a nucleus by the closest approach of alpha particles
To be able to calculate the radius of a nucleus by the diffraction angle of electrons
To be able to calculate the nuclear radius and nuclear density

Rutherford gave us an idea of the size of the nucleus compared to the atom but more experimental work has been done to find a more accurate measurement.



Closest Approach of Alpha Particles

Rutherford fired alpha particles at gold atoms in a piece of foil. They approach the nucleus but slow down as the electromagnetic repulsive force become stronger. Eventually they stop moving, all the kinetic energy has been converted into potential energy as the particles come to rest at a distance *r* from the centre of the nucleus.



$E_K = E_P$ → $E_P = qV$ where V is the electric potential at a distance of r from the centre

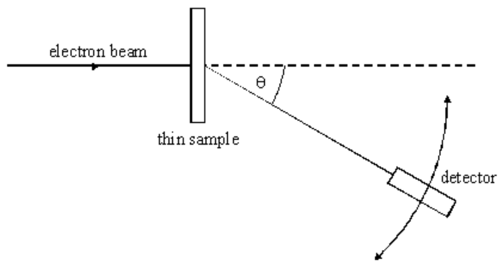
$$E_P = q\frac{Q}{4\pi\varepsilon_0 r} \rightarrow E_K = q\frac{Q}{4\pi\varepsilon_0 r} \rightarrow \boxed{r = q\frac{Q}{4\pi\varepsilon_0 E_K}}$$

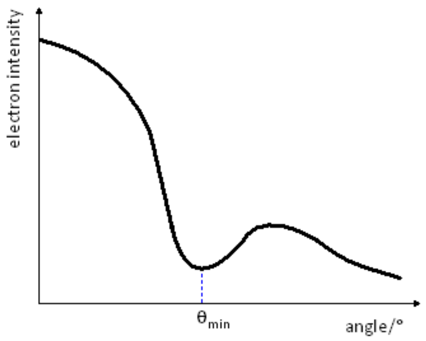
This gives us the upper limit of the radius of a nucleus.
Calculating the nuclear radius this way gives us a value of $r = 4.55 \times 10^{-14}$ m or 45.5 fm (where 1 fm = 1×10^{-15} m)
Modern measurements give us values of approximately r = 6.5 fm

(Remember that 1 eV of energy is equal to 1.6×10^{-19} J)

Electron Diffraction

A beam of electrons were fired at a thin sample of atoms and the diffraction pattern was detected and then examined.





The graph shows a minimum at a value of θ_{min}. We can use this to find a value of the nuclear radius.

$$\boxed{\sin\theta_{min} = \frac{0.61\lambda}{D}}$$

Where D is the nuclear radius and λ is the de Broglie wavelength of the beam of electrons. We can calculate this as follows:

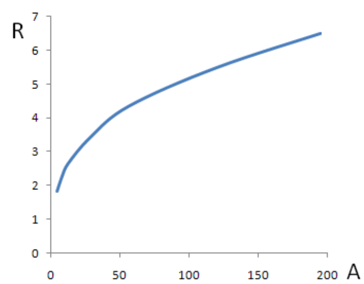
The kinetic energy gained by the electrons is $E_K = eV$ where e is the charge on the electron and V is the potential difference used to accelerate it. So we now have:

$$\tfrac{1}{2}mv^2 = eV \rightarrow mv^2 = 2eV \rightarrow m^2v^2 = 2meV \rightarrow \sqrt{m^2v^2} = \sqrt{2meV} \rightarrow mv = \sqrt{2meV}$$

We can now substitute this into the equation for de Broglie wavelength: $\lambda = \frac{h}{mv}$ → $\boxed{\lambda = \frac{h}{\sqrt{2meV}}}$

Nuclear Radius

From the experimental results a graph was plotted of R against A. A graph like the one to the right was obtained. They saw that R depends not on A, but on $A^{1/3}$.
When they plotted the graph of R against $A^{1/3}$ they found a straight line that cut the origin and had a gradient of r_0. (r_0 is a constant representing the radius of a single nucleon and has a value of between 1.2 and 1.5 fm)



The radius of a nucleus has been found to be: $R = r_0 A^{1/3}$

Nuclear Density

Now that we have an equation for the nuclear radius we can calculate the density of a nucleus.
If we have a nucleus of A nucleons, we can assume the mass is Au and the volume is the volume of a sphere:

$$\rho = \frac{m}{V} \quad \rightarrow \quad \rho = \frac{Au}{\frac{4}{3}\pi R^3} \quad \rightarrow \quad \rho = \frac{Au}{\frac{4}{3}\pi (r_0 A^{1/3})^3} \quad \rightarrow \quad \rho = \frac{Au}{\frac{4}{3}\pi r_0{}^3 A} \quad \rightarrow \quad \rho = \frac{u}{\frac{4}{3}\pi r_0{}^3}$$

We can see that the density is independent of the nucleon number and gives a value of: 3.4×10^{17} kg m^{-3}.