Kinetic Theory Equations

To be able to list the assumptions needed to derive an equation for the pressure of a gas
To be able to derive an equation for the pressure of a gas
To be able to calculate the mean kinetic energy of a gas molecule

Assumptions

1. There are a very large number of molecules (N)
2. Molecules have negligible volume compared to the container
3. The molecules show random motion (ranges of speeds and directions)
4. Newton's Laws of Motion can be applied to the molecules
5. Collisions are elastic and happen quickly compared to the time between collisions
6. There are no intermolecular forces acting other than when they collide

The Big, Bad Derivation

The molecules move in all directions. Let us start with one molecule of mass m travelling with velocity v_x. It collides with the walls of the container, each wall has a length of L.

Calculate the change in momentum: before it moves with velocity v_x and after the collision it move with $-v_x$.

$\Delta mv = (mv_x) - (-mv_x)$ → $\boxed{\Delta mv = 2mv_x}$ **Equation 1**

The time can be given by using distance/speed: the speed is v_x and the distance is twice the length of the box (the distance to collide and then collide again with the same wall) $\boxed{t = \dfrac{2L}{v_x}}$ **Equation 2**

Force can be calculated by: $F = \dfrac{\Delta mv}{\Delta t}$ Substitute in **Equation 1 and 2** → $F = \dfrac{2mv_x}{\left(\dfrac{2L}{v_x}\right)}$ → $F = 2mv_x . \left(\dfrac{v_x}{2L}\right)$

→ $\boxed{F = \dfrac{mv_x^{\ 2}}{L}}$ **Equation 3**, gives the force of <u>**one**</u> molecule acting on the side of the container.

We can now calculate the **pressure** this one molecule causes in the x direction:

$p = \dfrac{F}{A}$ Substituting **Equation 3** → $p = \dfrac{mv_x^{\ 2}/L}{L^2}$ → $p = \dfrac{mv_x^{\ 2}}{L^3}$ → $p = \dfrac{mv_x^{\ 2}}{V}$ **Equation 4**

(If we assume that the box is a cube, we can ***replace*** L^3 with V, both units are m^3)

All the molecules of the gas have difference speeds in the x direction. We can find the pressure in the x direction due to them all by first using the mean value of v_x and then multiplying it by N, the total number of molecules: $p = \dfrac{mv_x^{\ 2}}{V}$ → $p = \dfrac{m\overline{v_x^{\ 2}}}{V}$ $\boxed{p = \dfrac{Nm\overline{v_x^{\ 2}}}{V}}$ **Equation 5**

Equation 5 gives us the pressure in the <u>x direction</u>.

The mean speed in all directions is given by: → $\overline{c^2} = \overline{v_x^{\ 2}} + \overline{v_x^{\ 2}} + \overline{v_x^{\ 2}}$

But since the average velocities in all directions are equal: → $\overline{c^2} = 3\overline{v_x^{\ 2}}$, $\dfrac{\overline{c^2}}{3} = \overline{v_x^{\ 2}}$

We can substitute this into the **Equation 5** for pressure above:

$p = \dfrac{Nm\overline{v_x^{\ 2}}}{V}$ → $pV = Nm\overline{v_x^{\ 2}}$ → $pV = Nm\dfrac{\overline{c^2}}{3}$ → $\boxed{pV = \dfrac{1}{3}Nm\overline{c^2}}$ **Equation 6**

Kinetic Energy of a Gas

From the equation we have just derived we can find an equation for the mean kinetic energy of a gas:

Since $pV = \dfrac{1}{3}Nm\overline{c^2}$ and $pV = nRT$ combine these to get $\boxed{\dfrac{1}{3}Nm\overline{c^2} = nRT}$ **Equation 7**

Kinetic energy is given by $E_K = \frac{1}{2}mv^2$ so we need to make the above equation look the same.

$$\frac{1}{3}Nm\overline{c^2} = nRT \rightarrow \frac{1}{3}m\overline{c^2} = \frac{nRT}{N} \rightarrow m\overline{c^2} = \frac{3nRT}{N} \rightarrow \frac{1}{2}m\overline{c^2} = \frac{3nRT}{2N}$$

$$n = \frac{N}{N_A} \rightarrow N_A = \frac{N}{n} \rightarrow \frac{1}{N_A} = \frac{n}{N} \qquad \frac{1}{2}m\overline{c^2} = \frac{3RT}{2N_A}$$

Don't forget that cheeky chap Boltzmann $k = \frac{R}{N_A} \rightarrow \boxed{\frac{1}{2}m\overline{c^2} = \frac{3}{2}kT}$ **Equation 8**