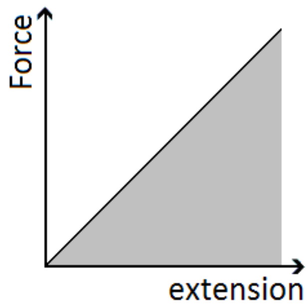
Hooke's Law and Elastic Strain energy

To be able to state Hooke's Law and explain what the spring constant is
To be able to describe how springs behave in series and parallel
To be able to derive the energy stored in a stretched material

Hooke's Law

If we take a metal wire or a spring and hang it from the ceiling it will have a natural, unstretched length of l metres. If we then attach masses to the bottom of the wire is will begin to increase in length (stretch). The amount of length it has increased by we will call the extension and represent by e.
If the extension increases proportionally to the force applied it follows Hooke's Law:
The force needed to stretch a spring is directly proportional to the extension of the spring from its natural length
So it takes twice as much force to extend a spring twice as far and half the force to extend it half as far.
We can write this in equation form: $F \propto e$ or $\boxed{F = ke}$
Here k is the constant that shows us how much extension in length we would get for a given force. It is called...

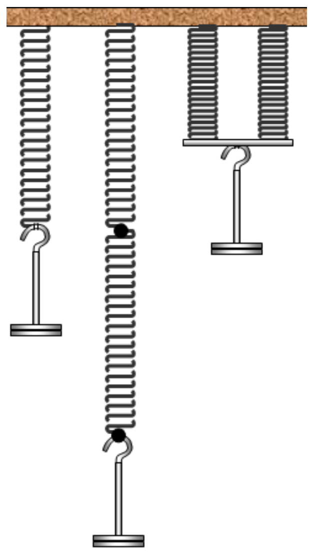
The Spring Constant



The spring constant gives us an idea of the stiffness (or stretchiness) of the material.
If we rearrange Hooke's Law we get: $k = \frac{F}{e}$
If we record the length of a spring, add masses to the bottom and measure its extension we can plot a graph of force against extension. The gradient of this graph will be equal to the spring constant.
A small force causes a large extension the spring constant will be *small – very stretchy*
A large force causes a small extension the spring constant will be *large – not stretchy*

Spring Constant is measured in Newtons per metre, N/m

Springs in Series



The combined spring constant of spring A and spring B connected in series is given by:
$\frac{1}{k_T} = \frac{1}{k_A} + \frac{1}{k_B}$ If A and B are identical this becomes:

$$\frac{1}{k_T} = \frac{1}{k} + \frac{1}{k} \rightarrow \frac{1}{k_T} = \frac{2}{k} \rightarrow k_T = \frac{k}{2}$$

Since this gives us a smaller value for the spring constant, applying the same force produces a larger extension. *It is stretchier*

Springs in Parallel

The combined spring constant of spring A and spring B connected in parallel is:
$k_T = k_A + k_B$ so if A and B are identical this becomes:

$$k_T = k + k \rightarrow k_T = 2k$$

Since this gives us a larger value for the spring constant applying the same force produces a smaller extension. *It is less stretchy*

Energy Stored (*Elastic Strain Energy*)

We can calculate the energy stored in a stretched material by considering the work done on it.
We defined work done as the force x distance moved in the direction of the force or $W = Fs$
Work done is equal to the energy transferred, in this case transferred to the material, so: $E = Fs$
The distance moved is the extension of the material, e, making the equation: $E = Fe$
The force is not constant; it increases from zero to a maximum of F. The average force is given by: $\frac{(F-0)}{2}$

If we bring these terms together we get the equation $E = \frac{(F-0)}{2}e$ which simplifies to: $\boxed{E = \tfrac{1}{2}Fe}$

This is also equal to the area under the graph of force against extension.

We can write a second version of this equation by substituting our top equation of $F = ke$ into the one above.