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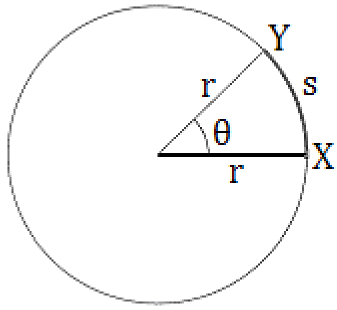
Circular Motion

To be able to calculate the angular displacement of an object moving in a circle
To be able to calculate the angular speed of an object moving in a circle
To be able to calculate the speed of an object moving in a circle

To the right is the path a car is taking as it moves in a circle of radius *r*.



Angular Displacement, θ

As the car travels from X to Y it has travelled a distance of *s* and has covered a section of the complete circle it will make. It has covered and angle of θ which is called the angular displacement.

$$\theta = \frac{arc}{radius}$$

$$\boxed{\theta = \frac{s}{r}}$$

Angular Displacement is measured in radians, rad

Radians

1 radian is the angle made when the arc of a circle is equal to the radius.

For a complete circle $\theta = \frac{arc}{radius}$ → $\theta = \frac{circumference}{radius}$ → $\theta = \frac{2\pi r}{r}$ → $\theta = 2\pi$

A complete circle is 360° so 360° = 2π rad

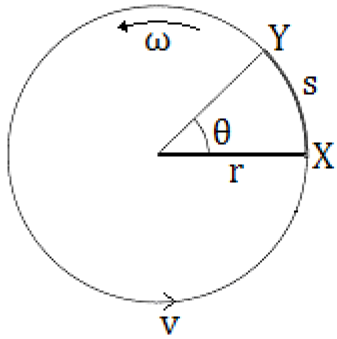
1° = 0.017 rad 57.3° = 1 rad

Angular Speed, ω

Angular speed is the rate of change of angular displacement, or the angle that is covered every second.

$$\boxed{\omega = \frac{\theta}{t}}$$

Angular Speed is measured in radians per second, rad/s or rad s^{-1}



Frequency, f

Frequency is the number of complete circles that occur every second.

For one circle; $\theta = 2\pi$, if we substitute this into the equation above we get

$$\omega = \frac{2\pi}{t}$$

This equation says that the angular speed (angle made per second) is equal to one circle divided by the time taken to do it. Very similar to speed = distance/time

Since $f = \frac{1}{T}$ the above equation can be written as $\boxed{\omega = 2\pi f}$

Frequency is measures in Hertz, Hz

Speed, v

The velocity of the car is constantly changing because the direction is constantly changing. The speed however, is constant and can be calculated.

$v = \frac{s}{t}$ If we rearrange the top equation we can get $r\theta = s$, the speed then becomes

$v = \frac{r\theta}{t}$ Now if we rearrange the second equation we get $\omega t = \theta$, the equation becomes

$v = \frac{r\omega t}{t}$ Cancel the *t*'s and we finally arrive at our equation for the speed.

$$\boxed{v = r\omega}$$