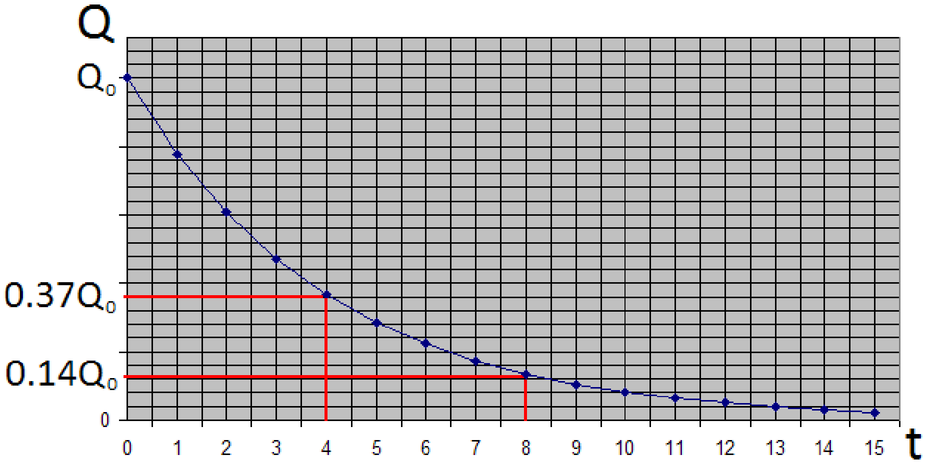
Exponential Decay

To be able to calculate the charge of a discharging capacitor after a time, t
To be able to calculate the potential difference across a discharging capacitor after a time, t
To be able to calculate the current through a discharging capacitor after a time, t

Finding τ from Graphs

The time constant of a discharging capacitor can be found from a graph of either charge, current or potential difference against time. After one time constant the value will have dropped to 0.37 of the initial value.



In this case the time constant is 4 seconds.

Quantitative Treatment

We could use the graph above to find the charge on the capacitor after a time, *t*. We could also use it to find the time it takes for the charge to fall to a value of *Q*.
This requires the graph to be drawn very accurately and values need to be taken from it very carefully.
Instead of doing this we can use the following equation to calculate the charge, *Q* after a time, *t*.

$$Q = Q_0 e^{-t/RC}$$

t is the time that has elapsed since discharge began
Q is the remaining charge
Q_0 is the initial (or starting) charge
RC is the time constant, also equal to the resistance multiplied by the capacitance.

Time is measured in seconds, s

When the time elapsed is equal to the time constant the charge should have fallen to 37% of the initial value.

$Q = Q_0 e^{-t/RC}$ → $Q = Q_0 e^{-RC/RC}$ → $Q = Q_0 e^{-1}$ (but e^{-1} = 0.37) → $Q = Q_0 0.37$

When the time elapsed is equal to twice the time constant the charge should have fallen to 37% of 37% of the initial value.

$Q = Q_0 e^{-t/RC}$ → $Q = Q_0 e^{-2RC/RC}$ → $Q = Q_0 e^{-2}$ (but e^{-2} = 0.37 x 0.37) → $Q = Q_0 0.14$

Similar equations can be established for the current flowing through and the potential difference across the capacitor after time, *t*:

$$\boxed{Q = Q_0 e^{-t/RC}} \qquad \boxed{I = I_0 e^{-t/RC}} \qquad \boxed{V = V_0 e^{-t/RC}}$$

Rearranging

The equations above can be rearranged to make *t* the subject. We will use the equation for charge:

$$Q = Q_0 e^{-t/RC} \rightarrow \frac{Q}{Q_0} = e^{-t/RC} \rightarrow \ln\left(\frac{Q}{Q_0}\right) = -t/RC \rightarrow \ln\left(\frac{Q}{Q_0}\right)RC = -t \rightarrow -\ln\left(\frac{Q}{Q_0}\right)RC = t$$

They can also be rearranged to make *RC* (time constant) the subject:

$$Q = Q_0 e^{-t/RC} \rightarrow \frac{Q}{Q_0} = e^{-t/RC} \rightarrow \ln\left(\frac{Q}{Q_0}\right) = {-t}/{RC} \rightarrow RC = {-t}\Big/{\ln\left(\frac{Q}{Q_0}\right)}$$